CIVE 440

Traffic Engineering and Simulation – Coordination part 2

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NETWORK COORDINATION

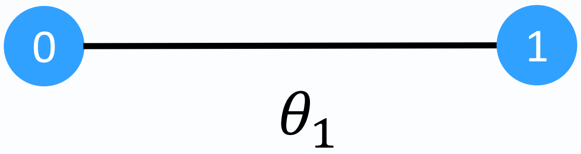
We've seen that coordinating a network in multiple directions is an optimisation problem with practical constraints on D_i, C, and v as well as the relationship between these.

- The primary problem is that putting traffic lights in a loop constraints the number of offsets that can be chosen between them (e.g. 3 offsets for 4 traffic lights).

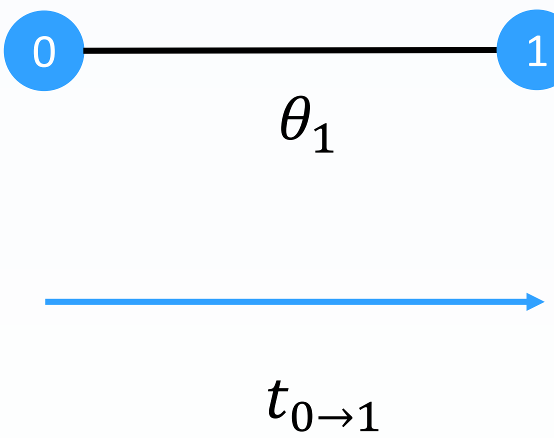
Stated more formally:

- between m traffic lights, there can exist at most $m - 1$ offsets between them (the first light acts as the master clock)

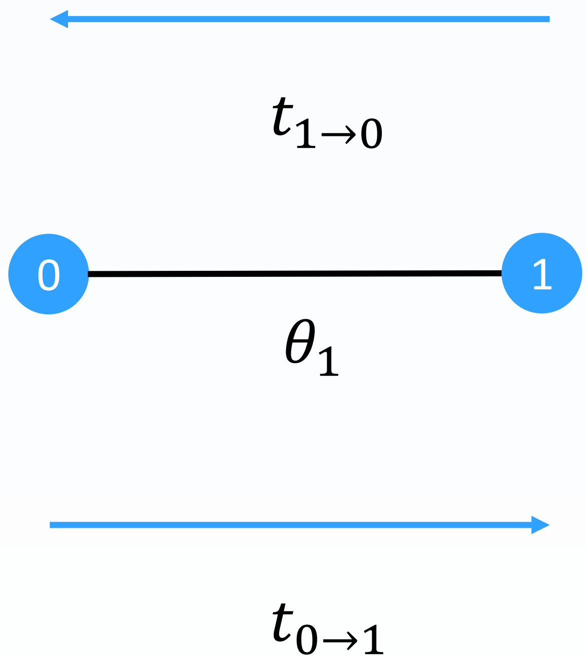
In this simple network, we have $m = 2$ lights and $m - 1 = 1$ offsets.



In simple coordination, we simply use travel time t_i between intersections as offsets.



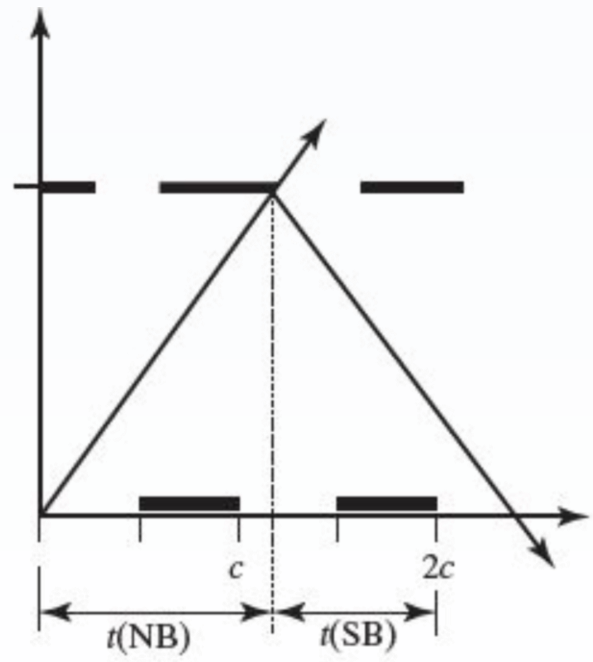
With multiple coordination, we now have competing travel times. Which to use?



Additionally, for an ideal solution, it's not enough for $t_{0\to 1} = t_{1\to 0}$. They also have to stay in phase, so we have an additional constraint (circuit constraint):

$$t_{0\to 1} + t_{1\to 0} = nC$$

where n is a whole number.



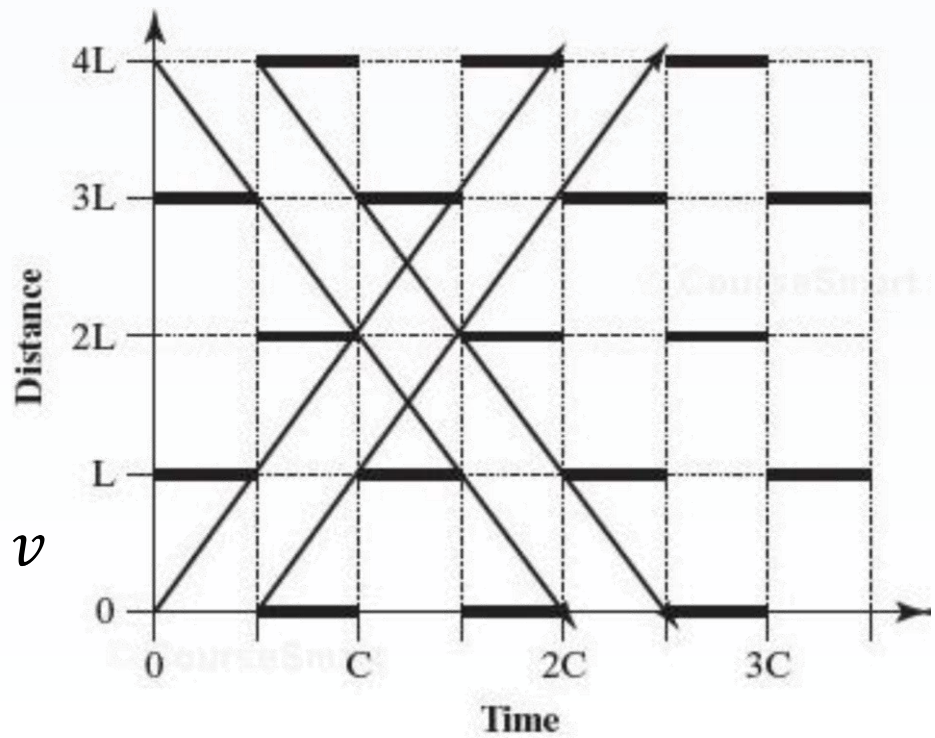
We previously briefly discussed some special cases of coordination that can optimise multiple directions:

- Alternating
- 2x alternating
- Simultaneous

ALTERNATING PROGRESSION

Intersections are alternatively in and out of phase with the master clock by $\theta = \frac{C}{2} = \frac{D}{v}$.

- For moderately spaced intersections
- There is no theoretical limit to number of signals that can be included.
- But important restrictions on choice of D_i, C and/or v

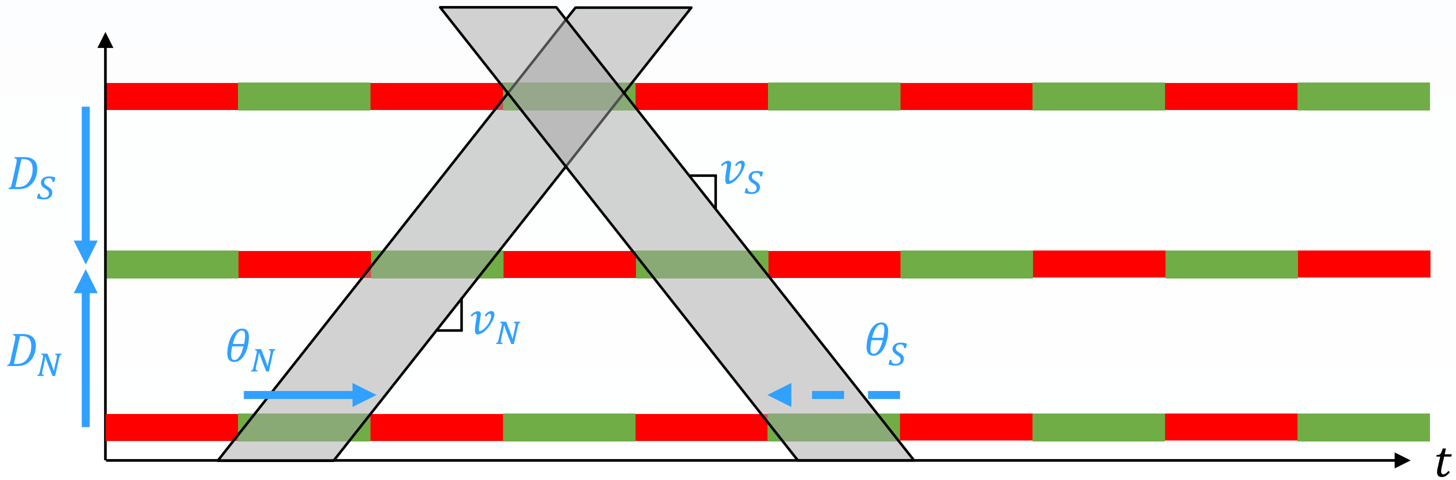


Ideal solution: $g_i = r_i$ (50/50 split), and choose D_i, C and/or v such that the travel time of both directions is in phase with the cycle length.

- Requires regular distances D_i and travel speeds v (or at least constant proportions between the two).

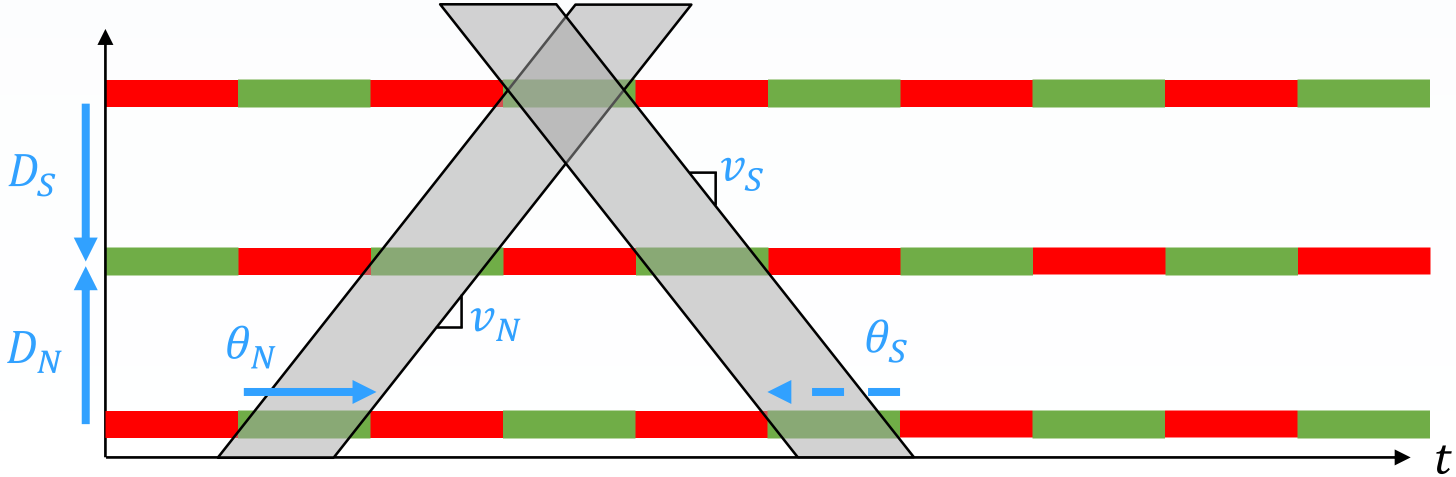
$$C = \theta_N + \theta_S$$

$$C = \frac{D_N \times 3.6}{v_N} - t_{qN} + \frac{D_S \times 3.6}{v_S} - t_{qS}$$



$$C = \frac{2D \times 3.6}{v} - t_{qN} - t_{qS}$$

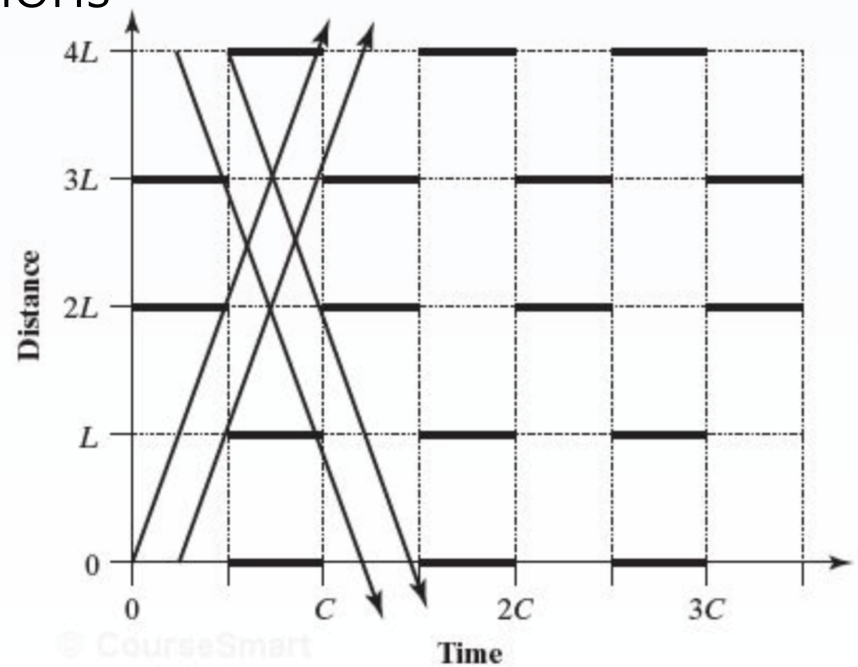
- With $v_{veh} = 50km/h$ and no queues:
  - a cycle of 30 seconds requires $D_i = 208m$
  - a cycle of 90 seconds requires $D_i = 625m$
- Parameter constraints:
  - D_i typically inflexible, enormous reconstruction costs
  - Loss of efficiency of green split calculation for isolated intersections
  - Design speed may or may not be flexible (safety versus comfort)



2X ALTERNATING PROGRESSION

Every two intersections are alternatively in and out of phase with the master clock by $\theta = \frac{C}{2} = \frac{D}{v}$.

- For closely spaced intersections
- There is no theoretical limit to number of signals that can be included.
- Similar constraints as alternating progression.
- Bandwidth is halved in both directions.

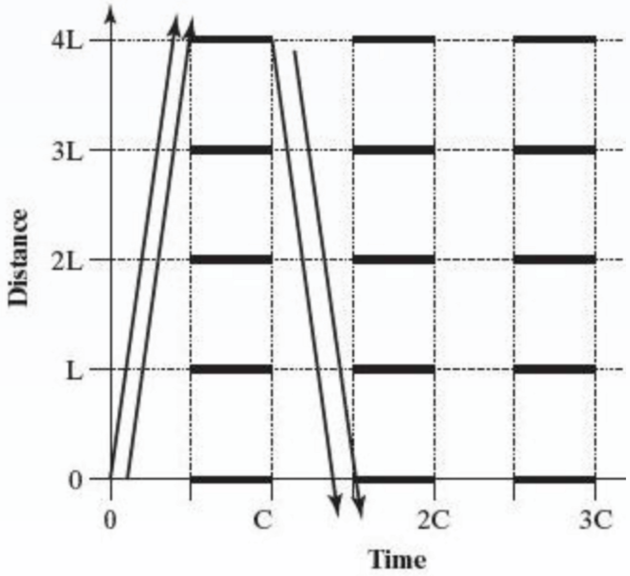


SIMULTANEOUS PROGRESSION

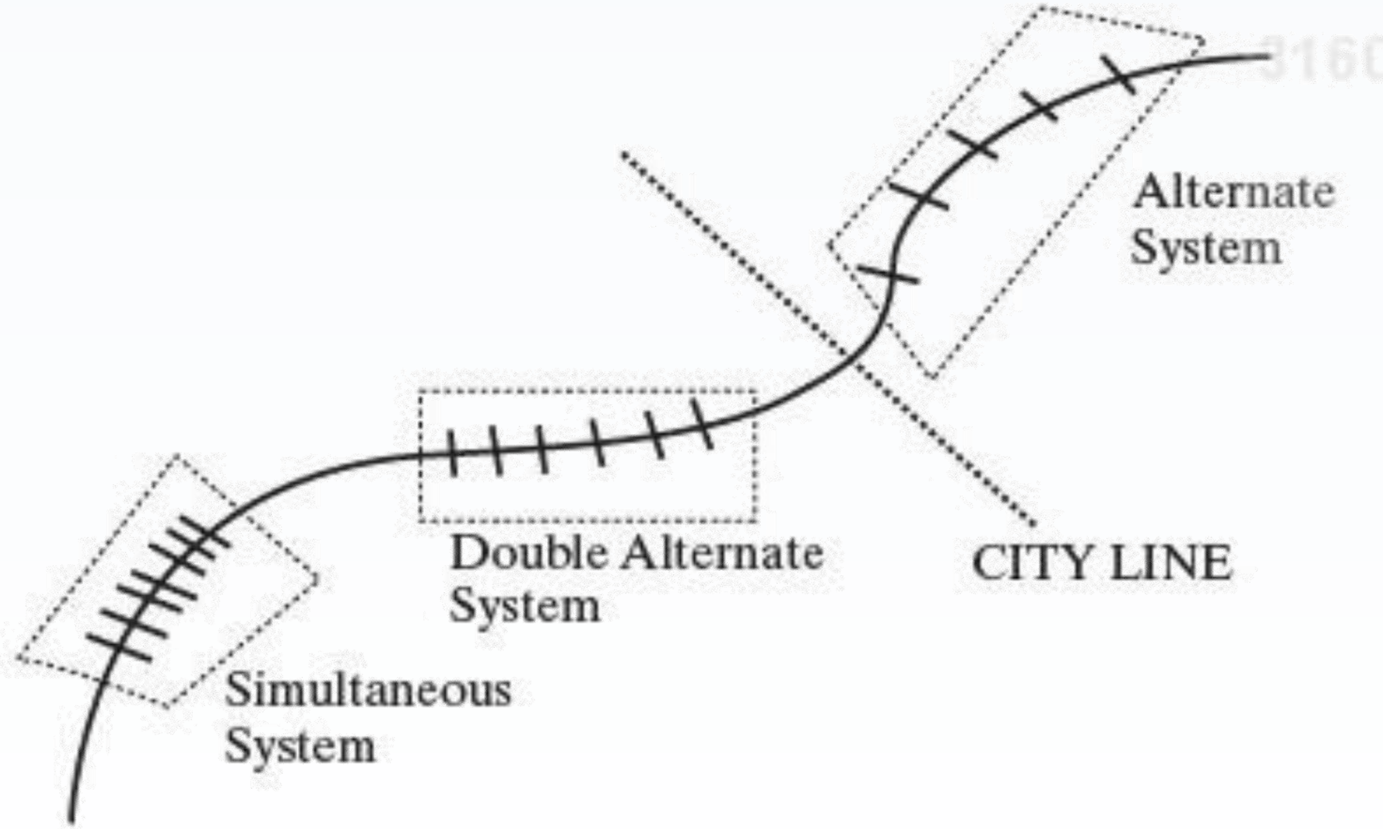
All intersections share the same clock (no offsets).

- For very closely spaced intersections m.
- No solution can be extended to infinity.

$$Efficiency = \left(\frac{1}{2} - \frac{(m-1)D}{sC}\right) 100\%$$



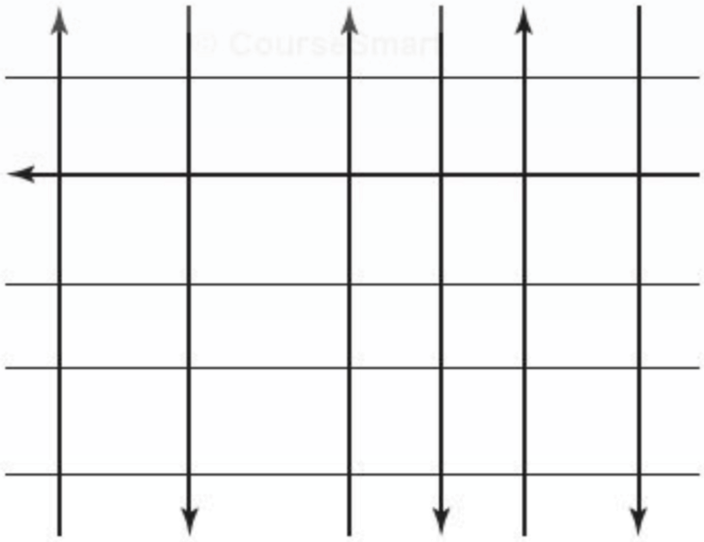
- Alternatively *very* distant intersections (i.e. ~600-1000 m).


Alternate
System
Double Alternate
System
CITY LINE
Simultaneous
System

MESHED NETWORKS

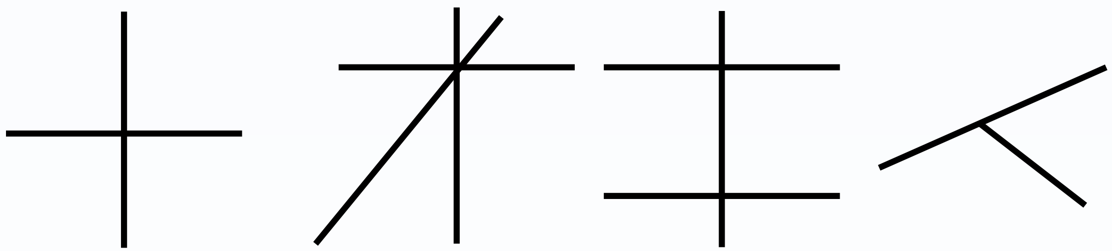
Network circuits are inevitable once we start coordinating enough streets, even with one-way coordination.

- A circuit occurs when a closed loop is formed from coordinated lights.
- Despite the name, vehicles are not necessarily driving in circles.
  - In fact, this has nothing to do with turning vehicles.
  - Instead we now include green time of alternate phase with travel time

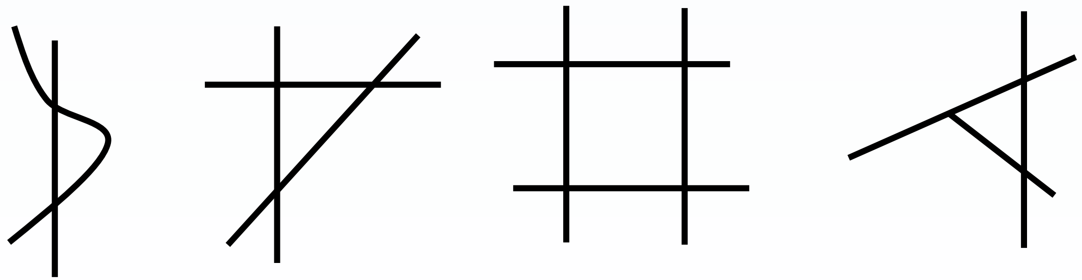


For meshed networks we can distinguish two distinct types:

- Open networks:

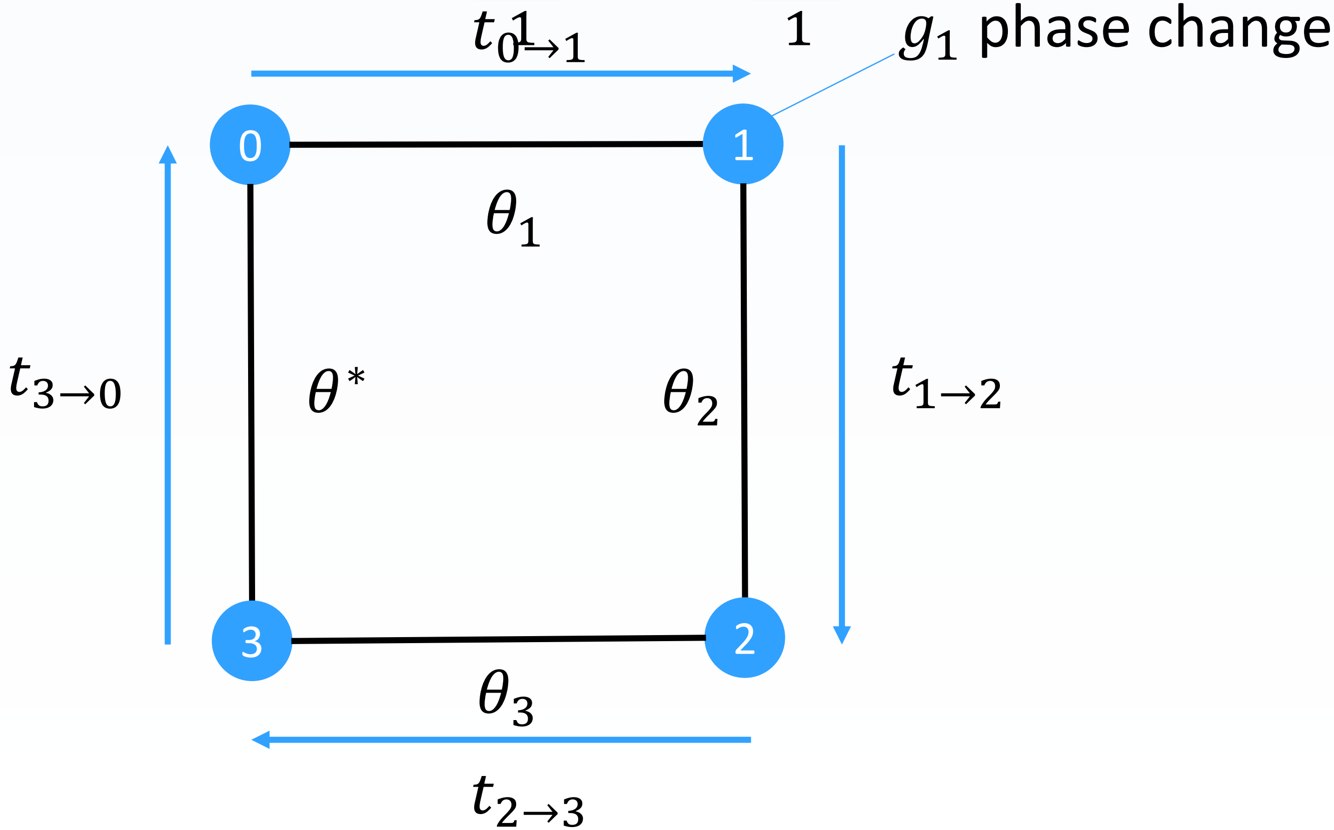


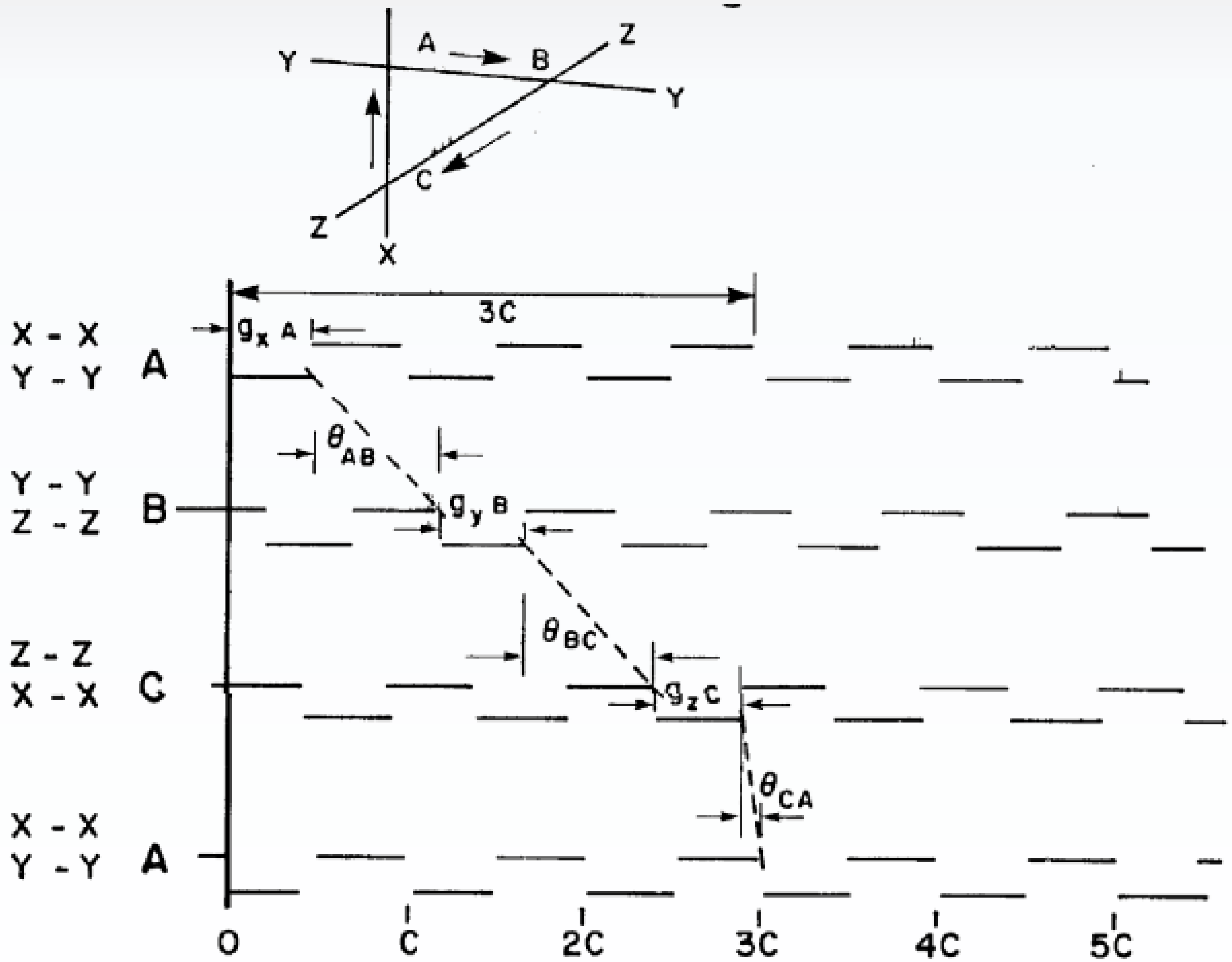
- Closed networks (circuit):



The same coordination constraints apply to network circuits more generally (for m intersections in a single circuit):

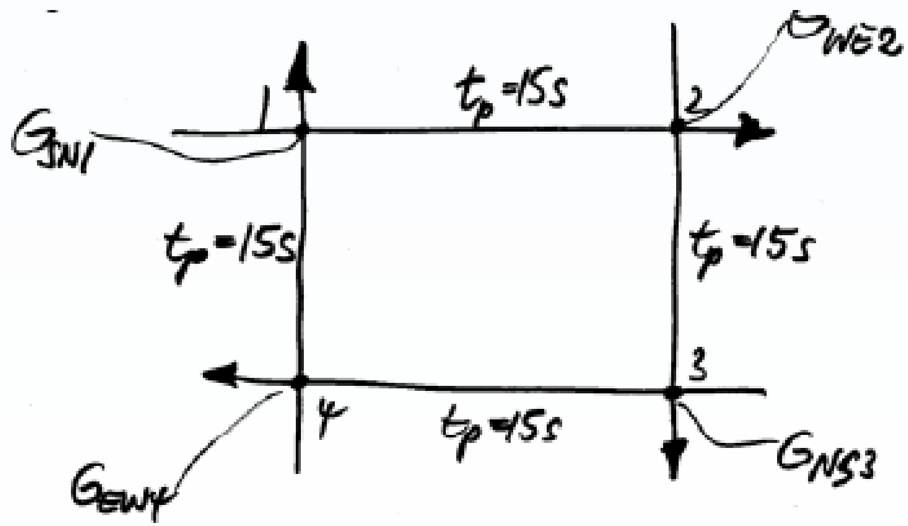
$$C > \theta^* = nC - \sum_{1}^{m-1} \theta_i - \sum_{1}^{m} g_i > 0$$




Y
A
B
Z
Y
C
Z
X
3C
X - X
Y - Y
A
g_x A
θ_{AB}
Y - Y
Z - Z
B
g_y B
θ_{BC}
Z - Z
X - X
C
g_z C
θ_{CA}
X - X
Y - Y
A
O
C
2C
3C
4C
5C

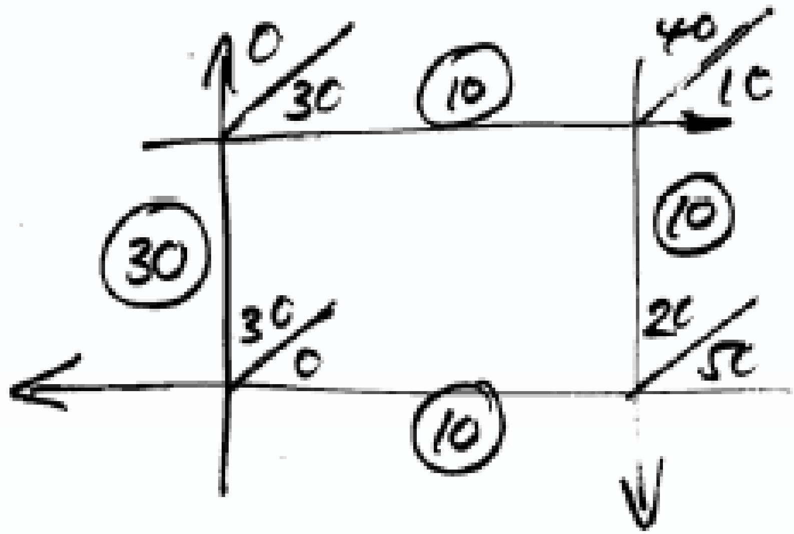
EXAMPLE

- One-way coordination in a loop
- 2 phase traffic lights with a 50/50 green split
- $C = 60s$



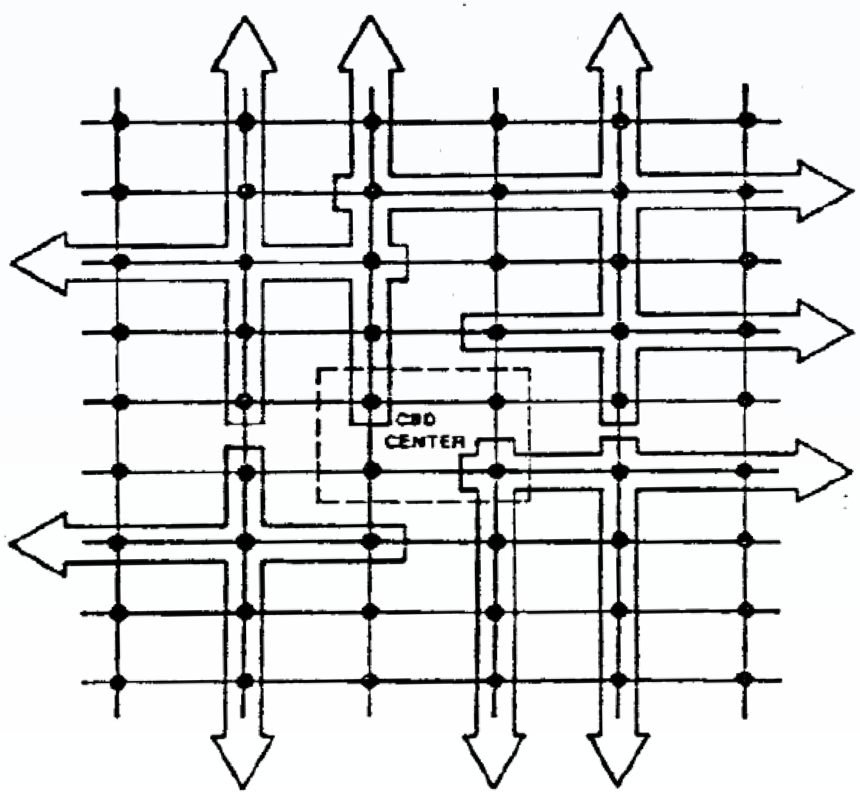
EXAMPLE

- Now with $t = 10s$



Large networks can be very complicated.

- One option is to avoid the problem altogether by limiting coordination to the primary directions of traffic flow and allow coordinated streets to not form loops. All of the intersections on the arrows below have the same cycle:

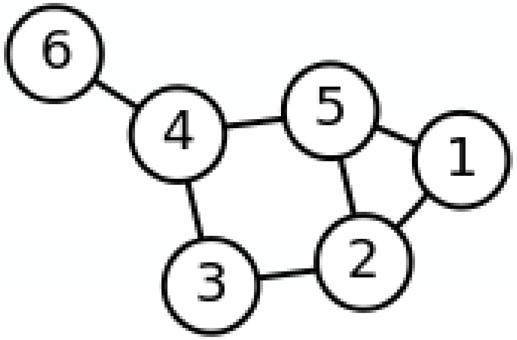


Preferential flow

GRAPH THEORY

As it turns out, there already exists a mathematical solution for solving this exact type of problem.

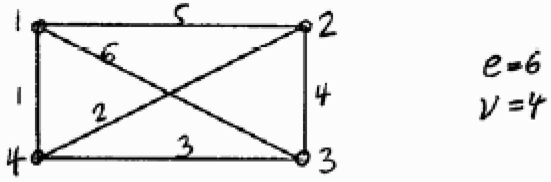
- Graph theory deconstructs closed networks into component trees to analyse circuit constraints.
- You may already be familiar with graph theory from electrical circuit calculations using Kirchhoff's circuit laws!
- Some graphs are non-oriented, while others are oriented. As we are dealing with a direction of flow, our graphs will be oriented!



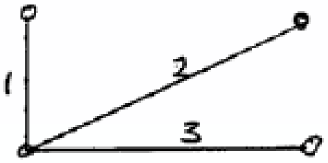
https://en.wikipedia.org/wiki/Graph_theory

Definitions:

- Vertices, v : Intersections being coordinated.
- Edges, e : Links between intersections (offsets from one to the other).

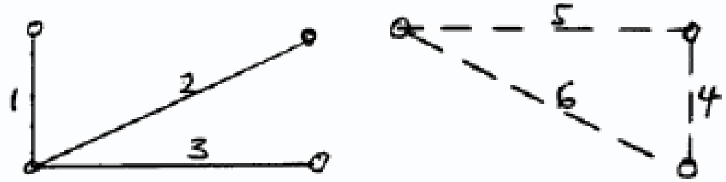


- Incidence : Vertex with a particular edge connected to it, connecting it to another vertex.
- Tree: Sub graph containing all vertices and the minimum number of edges to connect all vertices together.

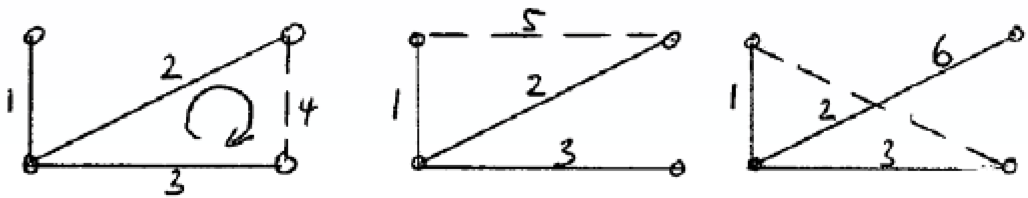


Definitions:

- Branches, b : Edges of the tree, each of which provides a degree of freedom. There are $b = v - 1$ branches.
- Degree of a vertex: The number of branches that are incident with that vertex.
- Cord, c : Edge corresponding to the complement of the tree (those edges that are not branches). There are $c = e - v + 1$ cords for any given tree.



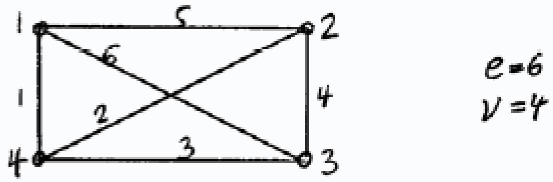
- Circuit : Closure of the tree when a cord is attached to it.
- Fundamental circuit, c': $c' = c$ circuits formed by the successive addition of each cord.



Definitions:

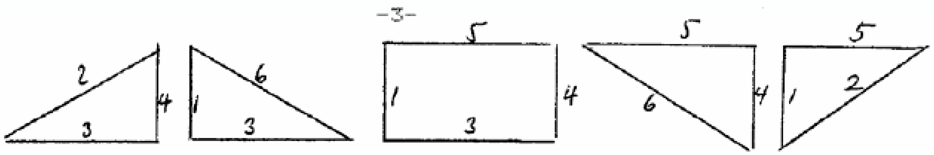
- Incidence matrix $\boldsymbol{A}$ (size $v \times e$) : $a_{i,j} = 1$ if the edge j is incident to the vertex i, $a_{i,j} = 0$ otherwise. $a_{i,j} = -1$ if the orientation differs.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$



- Incidence matrix $\boldsymbol{B}$ (size $c \times e$) : $b_{i,j} = +1$ if the edge j is part of circuit c' and conserves its orientation, $b_{i,j} = -1$ if the orientation is not conserved and $b_{i,j} = 0$ otherwise.

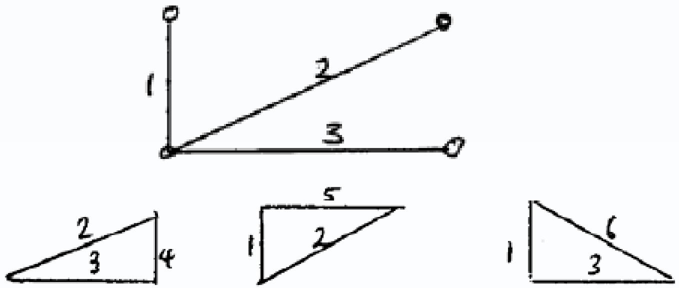
$$B = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$



Definitions:

- Fundamental circuit matrix $\boldsymbol{B_f}$ (size $c' \times (v-1) * 2$) : contains two matrices
  - One identity matrix (size $c' \times (v-1)$).
  - One incidence matrix limited to the fundamental circuits only (c') and to the branches $(v-1)$ of the primary tree.

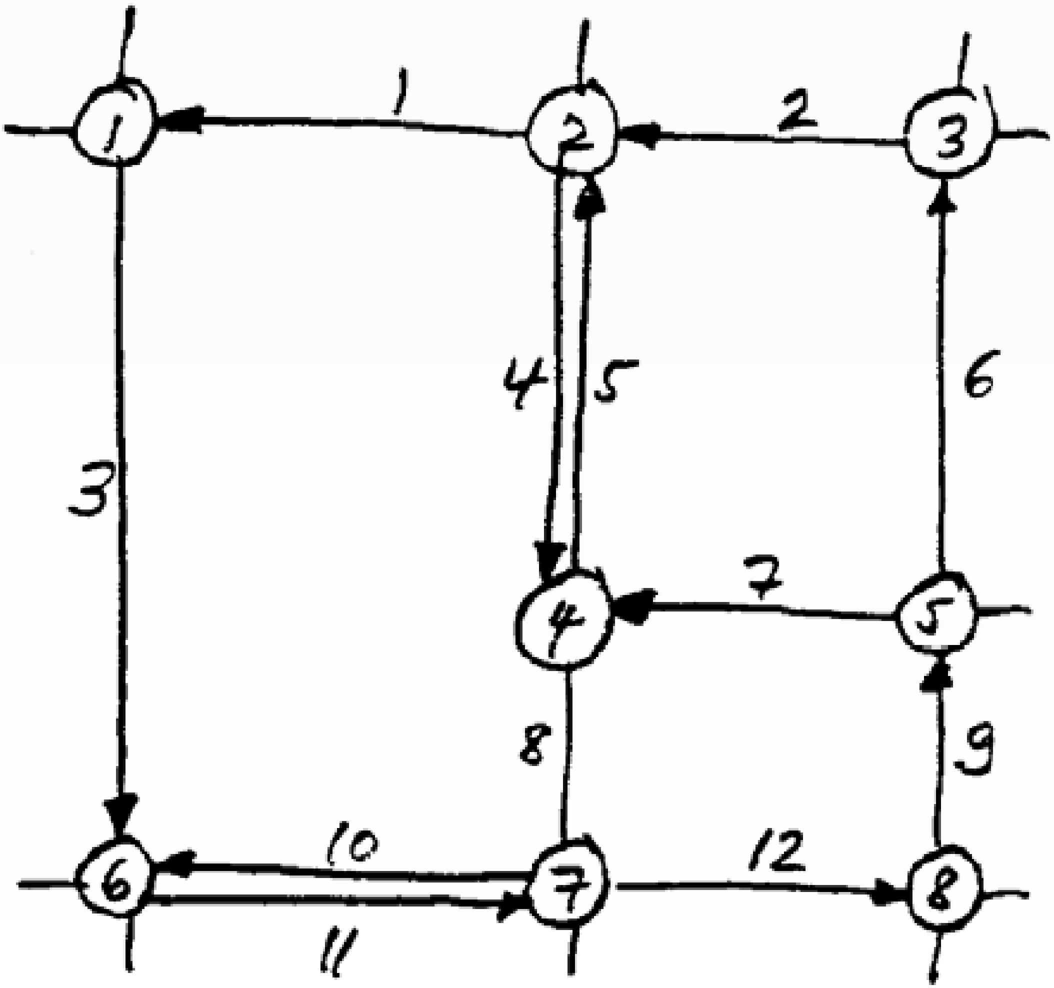
$$B_f = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$



These graphs are oriented for the purpose of traffic analysis!

EXAMPLE NETWORK

$e = 12$
$v = 8$
$c = e - v + 1 = 5$
$c' = 9$



Recall:

$$C > \theta^* = nC - \sum_{1}^{n-1} \theta_j - \sum_{1}^{n} g_j > 0$$

Stated more formally, we have the following circuit constraint for circuit i with edge j:

$$\sum \theta_j + \sum g_j = n_i C$$

Representing incidence (closure of circuit) with a direction of coordination at a green light $g_{i,j-1}$ with the multiplication of θ_j with $b_{i,j}$:

$$\sum b_{i,j}\theta_j + \sum c_{i,j} g_j = n_i C$$

This sets us up for a list of $i = 1,2,\ldots$ constraints, one for each circuit in the network. Variable space:

$$\vec{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_e \end{bmatrix} \quad \vec{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_c \end{bmatrix}$$

Our previous equation thus becomes, in matrix notation:

$$\boldsymbol{B}\vec{\theta} + \boldsymbol{C}\vec{g} = \vec{n}C$$

Also, we have $b = v - 1$ degrees of freedom and $c = e - v + 1$ constraints.

- Therefore, let us limit the number of solutions to the minimum number of fundamental circuits $c\,'$ in the network $\boldsymbol{B}_f$:

$$\boldsymbol{B}_f \vec{\theta} + \boldsymbol{C}_f \vec{g} = \vec{n} C$$

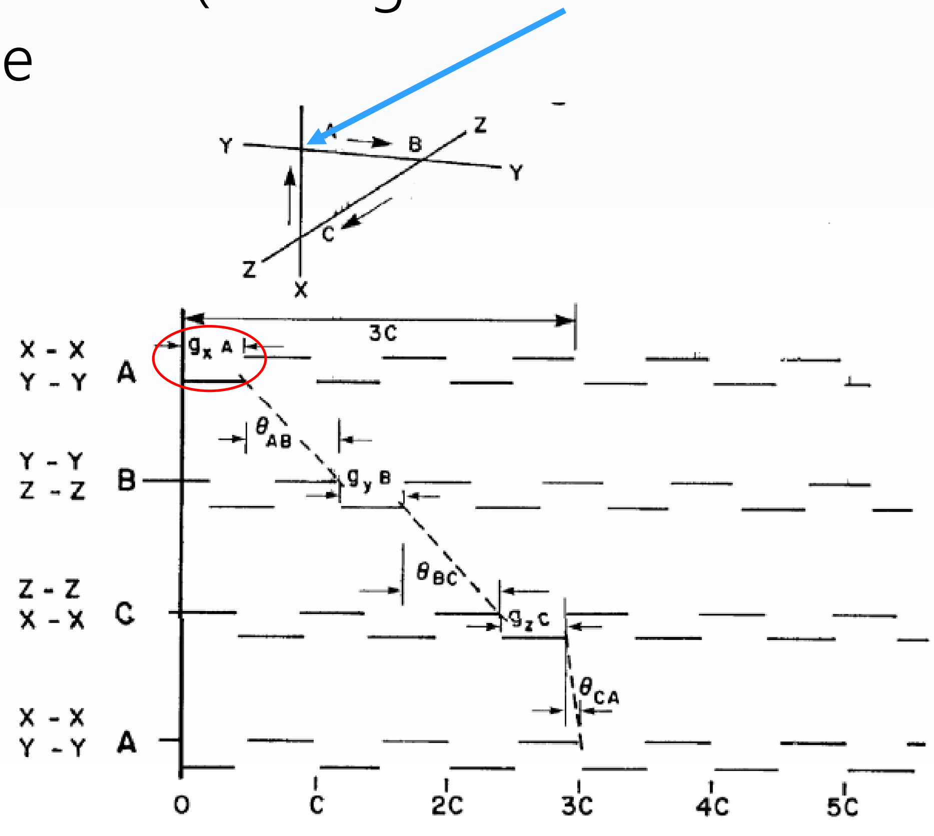
- We will also have to rearrange the offset vector to separate those offsets that can be chosen freely and those that cannot :

$$\vec{\theta} = \left[\frac{\vec{\theta}_c}{\vec{\theta}_b} \right]$$

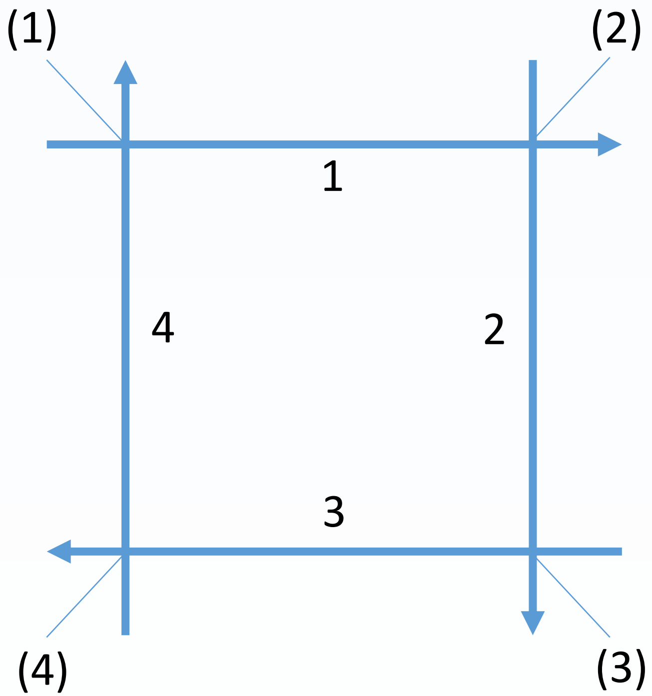
- The choice of n_i has no impact on the solution since the offsets only affect relative phasing.

$\boldsymbol{C}$ is a similar incidence matrix, but follows a different rule according to traffic light phases:

- For each edge, following the orientation of the circuit, the value of $c_{i,j}$ is 1 if the next edge is out of phase at the intersection between the two (i.e. angle of $90°$ or $270°$.
- The value is 0 otherwise or if the intersection j is not part of circuit i.



EXAMPLE

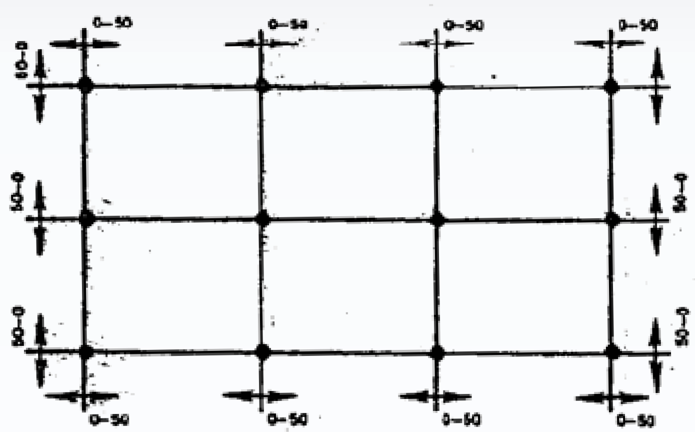


$$C = 50s$$

$$\frac{g}{C} = 50\%$$

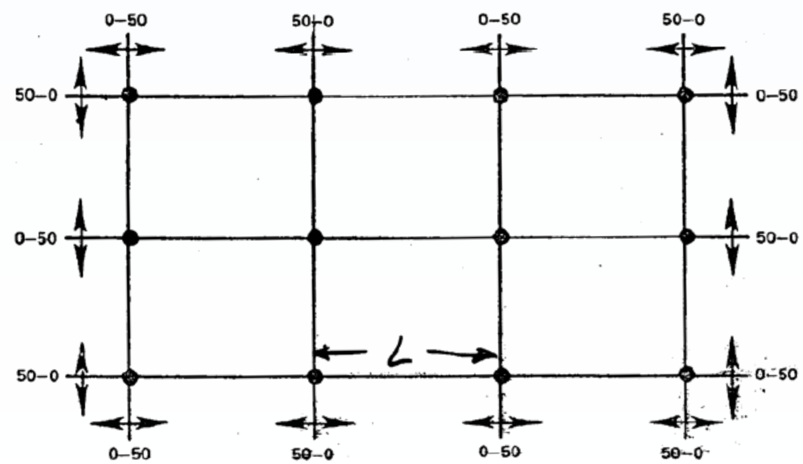
$$t_{1\to 2} = t_{2\to 3} = t_{3\to 4} = t_{4\to 1} 10s$$

Simultaneous coordination (by axis):



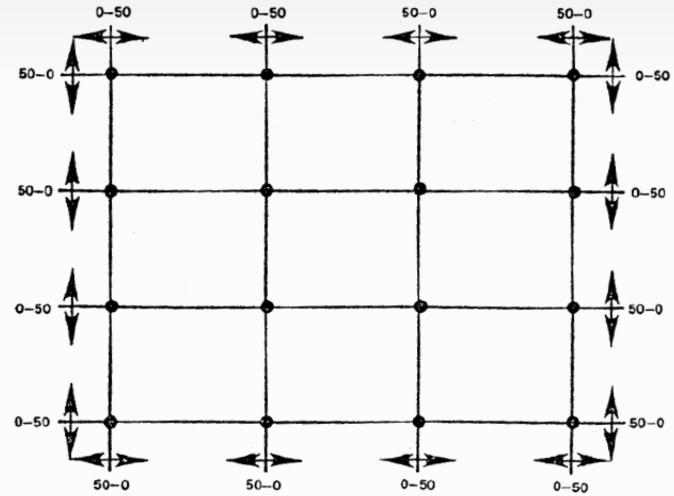
Simultaneous operation

Alternating coordination:



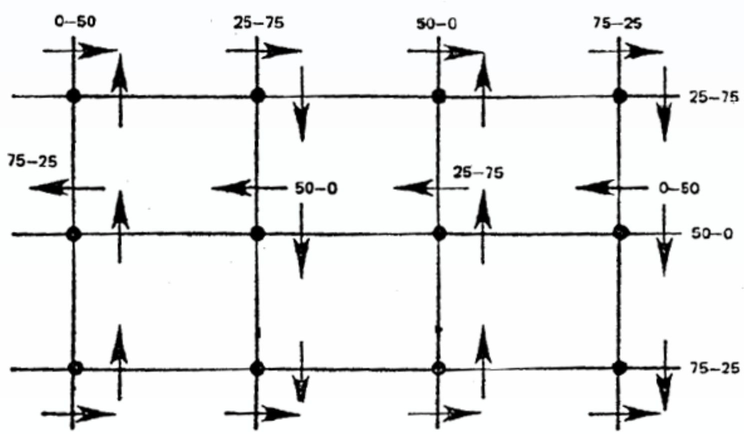
Single-alternate timing

2x alternating coordination:



Double-alternate timing

Offset coordination with C/4:



Quarter-cycle offset timing

That's all for today!